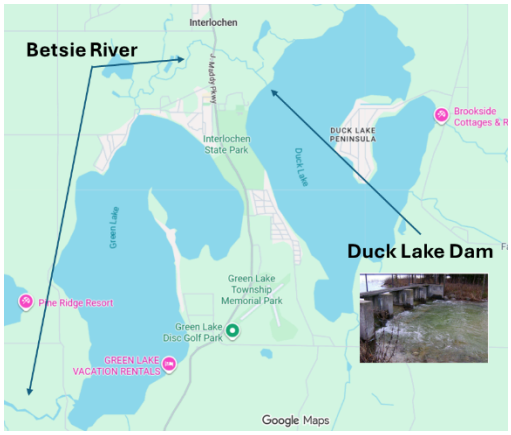
Statistical Analysis of Duck and Green Lake Levels

Submitted by Duck Lake Riparian, Bruce Corner

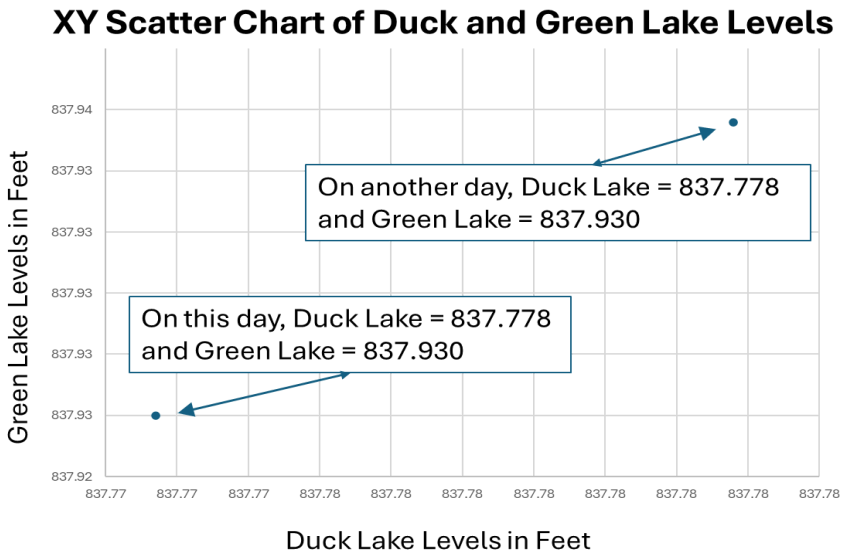
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Duck Lake and Green Lake are neighboring twin lakes located in Green Lake Township, Grand Traverse County, Michigan. Duck Lake is the headwater of the Betsie River and flows into Green Lake via a dam which partially controls Duck Lake levels. In order to protect Duck Lake shorelines from ice erosion, the Duck Lake Dam is manipulated to decrease lake levels in the Fall and Winter and increase for the Spring and Summer seasons.

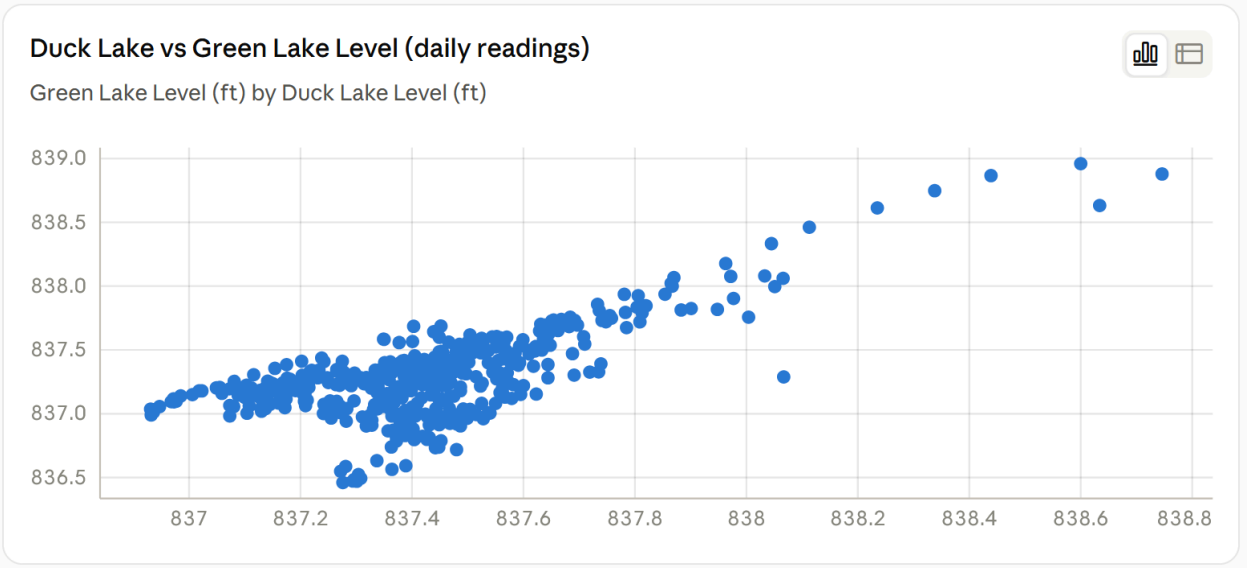


A study of Duck and Green Lake levels has been conducted (where both lake levels have been recorded on the same days). It started on November 7, 2022 and is on-going. Some have hypothesized that the Duck Lake dam and its level manipulations have a detrimental effect on Green Lake levels. Since Green Lake has other sources of water, it has not been clear if this hypothesis is true. I secured the lake level data from the Grand Traverse County Drain Commission and used my professional background in statistical analysis to review it. In this document, I will show that the statistical evidence of lake level changes is more consistent with a common cause (shared basin, weather, etc.) rather than with Duck Lake mechanically causing changes in Green Lake.

One way to show the relationship between the two lake levels is to plot them in an "XY Scatter Plot." The example plot immediately below shows how to depict each day's lake levels. Each point represents BOTH lake levels for one day.



The following lake level chart covers **1,411 daily readings from Nov 7, 2022 to Sep 14, 2026**, with a handful of gaps (4 missing Duck Lake values, 8 missing Green Lake values).



Visual Inspection:

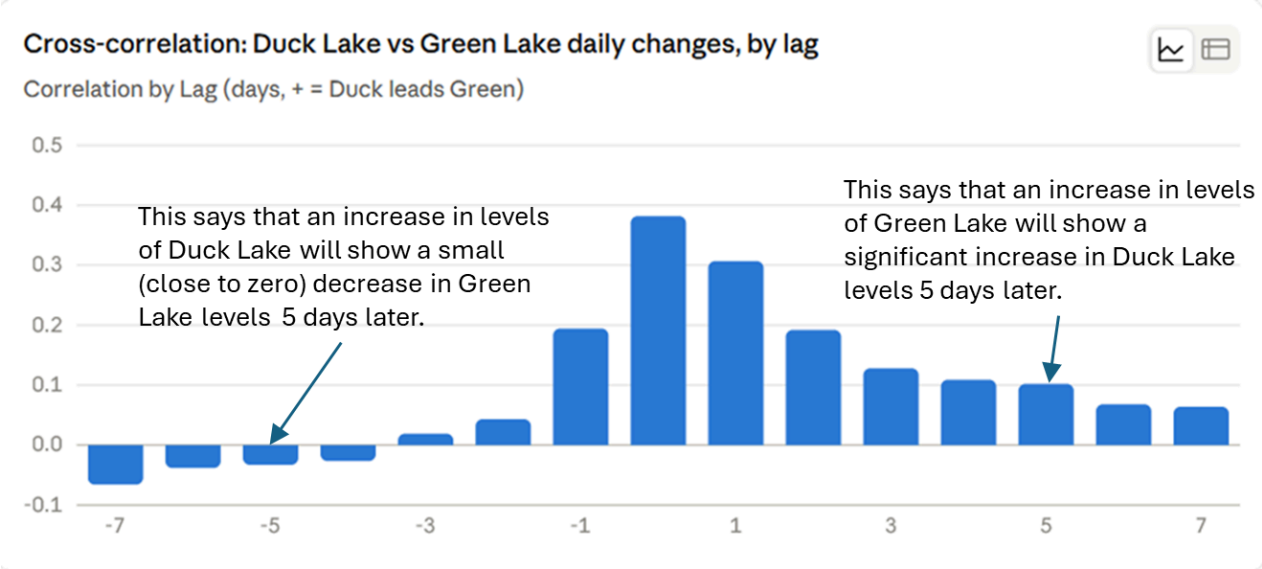
- These data show Duck Lake levels across the horizontal X-Axis and the Green Lake data plotted along the vertical Y-Axis. Each point represents one day.
- Note that the plot points spread from lower left to upper right. So, if Duck Lake levels are high, so are Green Lake levels. If one is low, they both are low. This indicates a positive correlation. If Duck Lake were manipulating levels to the detriment of Green

Lake, we would expect a negative correlation with an overall shape of upper left to lower right. This is because we would expect Green Lake to fall when Duck Lake is increasing.

More Precise Correlation results:

- **Pearson r = 0.71** — This indicates a strong, positive linear correlation between the two lakes' levels
- **Spearman ρ = 0.56** — This shows a moderate rank correlation (lower than Pearson, suggesting the relationship isn't perfectly monotonic/linear throughout the range — likely due to some regime shifts, like the cluster of low Green Lake readings around 836.8–837.0 ft where Duck Lake barely moves and is not increasing.)
- **Correlation of day-to-day changes = 0.38** — When you look at daily *fluctuations* rather than raw levels, the relationship weakens noticeably. Studying the fluctuations is a more precise calculation for causal links between the two lakes. This is because we are looking at relating the changes in Duck Lake to changes in Green Lake. This tells you the strong r=0.71 is driven largely by both lakes trending together over the long run (e.g., seasonal cycles), rather than the two lakes reacting to each other tick-for-tick, day to day.
- **Statistical "Granger causality" test** (which asks, "Does knowing Duck Lake's past changes help predict Green Lake's future changes, beyond what Green Lake's own history tells you?") Instead of relating to the lake levels on the same day, the Granger test looks at lead/lag relationships. Example: Do changes in Duck Lake levels on day 1 correlate with changes in Green Lake 5 days later?

    - Duck to Green Causality: This is statistically significant ($p < 0.0001$ at 1-day lag). Duck's changes do have some predictive power for Green. And, intuitively we should expect some causality. If Duck were to increase the dam level, we would expect Duck to raise while Green Lake to fall for at least a short period.
    - Green to Duck Causality: This is also significant, and noticeably <u>stronger</u> (larger F-statistics at every lag tested).
    - That bidirectional result is the key finding. If Duck Lake were mechanically driving Green Lake (e.g., water flowing from one to the other), you'd typically expect prediction to run mainly in one direction. Instead, each lake helps predict the other, with <u>Green actually having the stronger edge</u> — a classic sign that **both are being pushed by the same underlying force** (rainfall/snowmelt, a shared watershed or groundwater table, or coordinated water-management releases) rather than actions on one lake directly causing results on the other.
    - **Cross-correlation of daily changes by lag** also supports this: the correlation peaks at lag 0 (same day, r = 0.38) and decays fairly symmetrically going backward and forward — it doesn't show the sharp "leads by X days" signature you'd expect from actual water transfer between the lakes.



Bottom line: The two lakes move together strongly, but the statistical evidence is more consistent with a common cause (shared basin, weather, or water management) than with Duck Lake mechanically causing changes in Green Lake. Granger causality tests statistical predictability, not true physical causation. And, as mentioned above, common sense does suggest that because of the twice per year changes in the dam, a small amount of causation is warranted. However, it is not statistically significant.